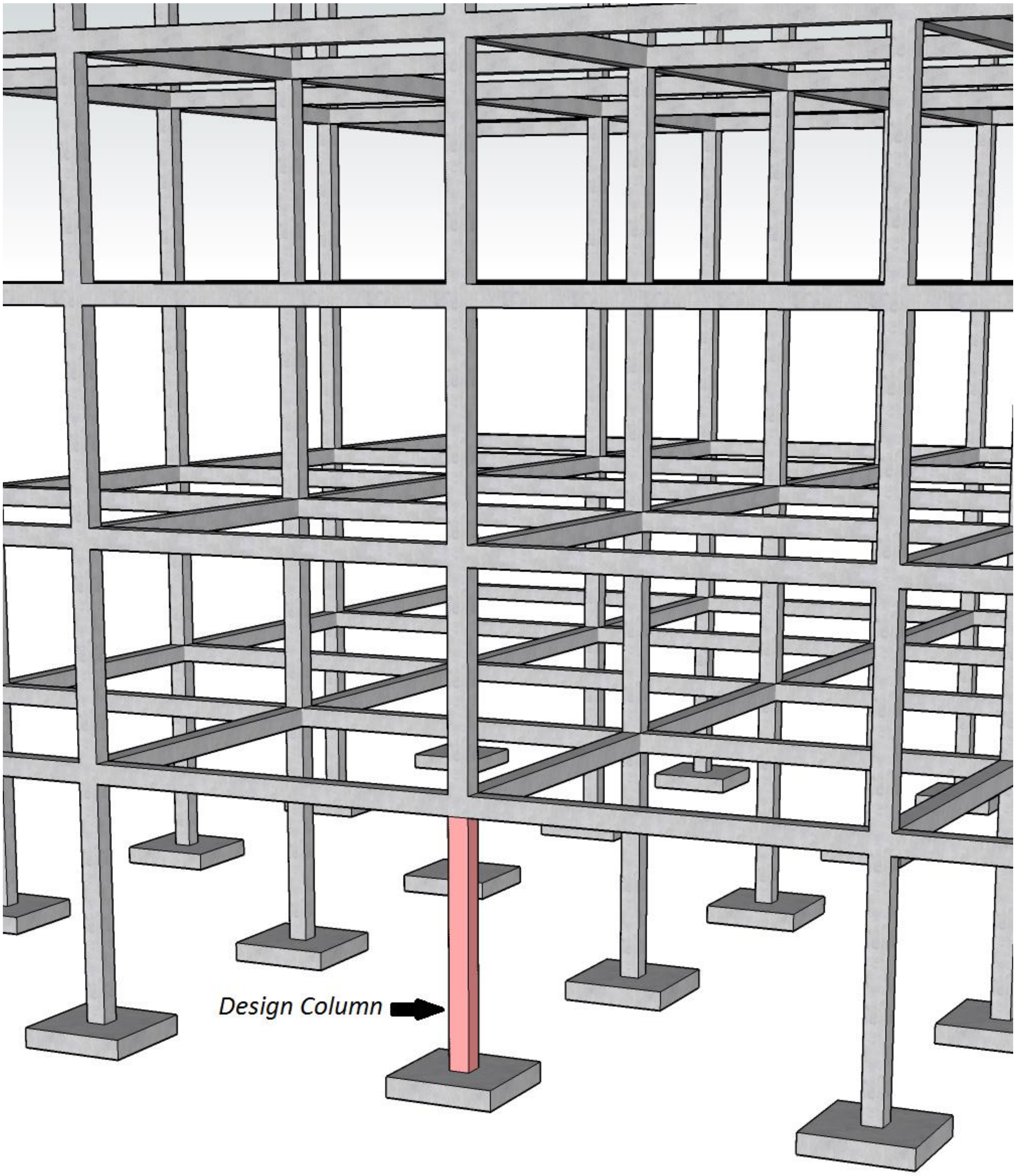
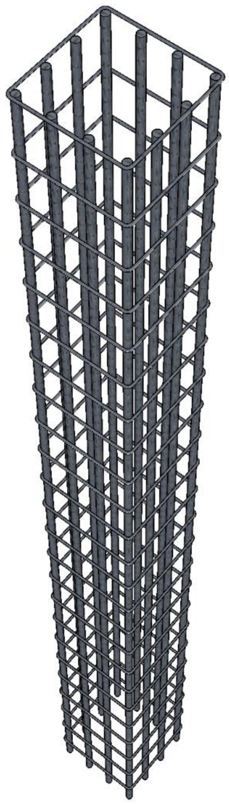
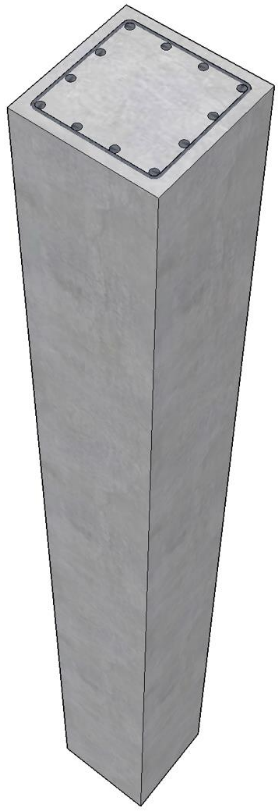
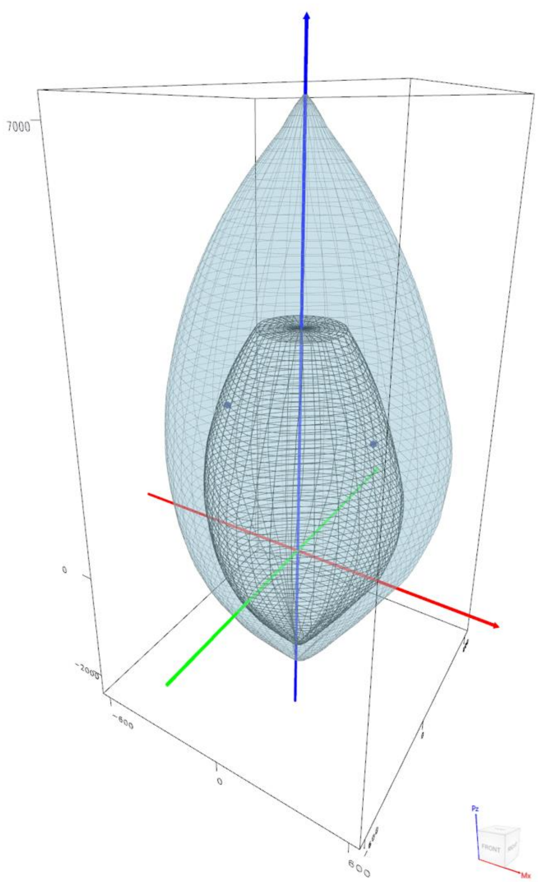
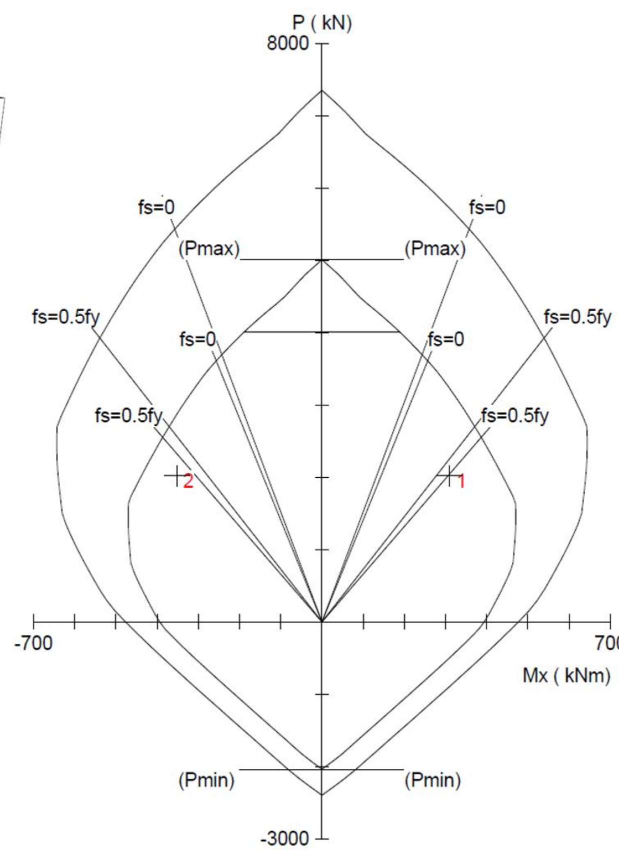
Slenderness Effects for Concrete Columns in Sway Frame - Moment Magnification Method (CSA A23.3-04)








7000
0


P (kN)
8000
fs=0
(Pmax)
fs=0
(Pmax)
fs=0.5fy
fs=0
fs=0
fs=0.5fy
fs=0.5fy
fs=0.5fy
2
1
-700
700
Mx (kNm)
(Pmin)
(Pmin)
-3000

Slender Concrete Column Design in Sway Frame Buildings

Evaluate slenderness effect for columns in a sway frame multistory reinforced concrete building by designing the first story exterior column. The clear height of the first story is 4.75 m, and is 2.75 m for all of the other stories. Lateral load effects on the building are governed by wind forces. Compare the calculated results with the values presented in the Reference and with exact values from spColumn engineering software program from StructurePoint.

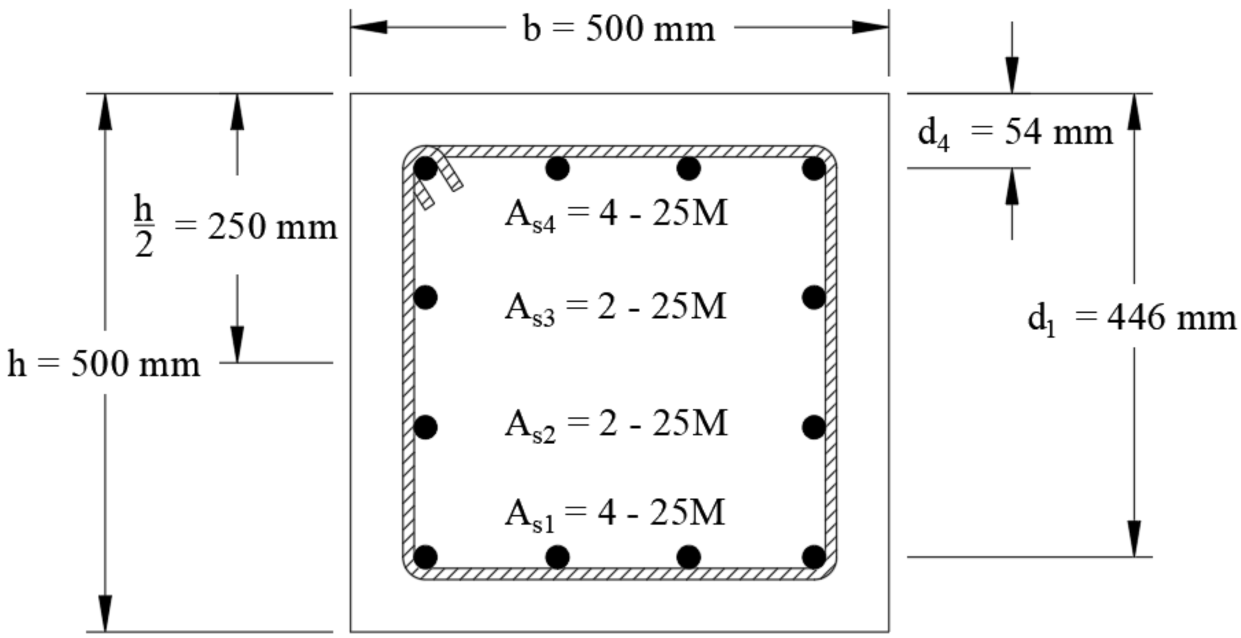


Figure 1 – Slender Reinforced Concrete Column Cross-Section

Contents

Code

Design of Concrete Structures (CSA A23.3-04)

Explanatory Notes on CSA Standard A23.3-04

Reference

Reinforced Concrete Mechanics and Design, First Canadian Edition, 2000, James MacGregor and Michael Bartlett, Prentice Hall, Example 12-3, 4 and 5.

Notes: Reference examples are based on CSA A23.3-94

This example is solved using CSA A23.3-04

Design Data

f_c' = 25 MPa for columns

f_y = 400 MPa

Slab thickness = 180 mm

Exterior Columns = 500 mm × 500 mm

Interior Columns = 500 mm × 500 mm

Interior Beams = 450 mm × 750 mm × 9 m

Exterior Beams = 450 mm × 750 mm × 9.5 m

Total building loads in the first story from the reference:

Table 1 – Total building factored loads

CSA A23.3-04 Reference	No.	Load Combination	$\sum P_f$, kN
Annex C Table C.1	1	1.4D	66,640
	2	1.25D + 1.5L	77,500
	3	1.25D + 1.5L + 0.4W	77,500
	4	1.25D + 1.5L - 0.4W	77,500
	5	0.9D + 1.5L + 0.4W	60,840
	6	0.9D + 1.5L - 0.4W	60,840
	7	1.25D + 0.5L + 1.4W	65,500
	8	1.25D + 0.5L - 1.4W	65,500
	9	0.9D + 0.5L + 1.4W	48,840
	10	0.9D + 0.5L - 1.4W	48,840

1. Factored Axial Loads and Bending Moments

1.1. Service loads

Table 2 - Exterior column service loads			
Load Case	Axial Load, kN	Bending Moment, kN.m	
		Top	Bottom
Dead, D	1,615.2	-107.36	-118
Live, L	362.86	-67.43	-72.86
Wind, W	0	-90.19	-105.33

1.2. Load Combinations – Factored Loads

CSA A23.3-04 (Annex C, Table C.1)

Table 3 - Exterior column factored loads									
CSA A23.3-04 Reference	No.	Load Combination	Axial Load, kN	Bending Moment, kN.m		$M_{Top,ns}$ kN.m	$M_{Bottom,ns}$ kN.m	$M_{Top,s}$ kN.m	$M_{Bottom,s}$ kN.m
				Top	Bottom				
Annex C Table C.1	1	1.4D	2,261	150.3	165.2	150.3	165.2	0.0	0.0
	2	1.25D + 1.5L	2,563	235.3	256.8	235.3	256.8	0.0	0.0
	3	1.25D + 1.5L + 0.4W	2,563	271.4	298.9	235.3	256.8	36.1	42.1
	4	1.25D + 1.5L - 0.4W	2,563	199.3	214.7	235.3	256.8	-36.1	-42.1
	5	0.9D + 1.5L + 0.4W	1,998	233.8	257.6	197.8	215.5	36.1	42.1
	6	0.9D + 1.5L - 0.4W	1,998	161.7	173.4	197.8	215.5	-36.1	-42.1
	7	1.25D + 0.5L + 1.4W	2,200	294.2	331.4	167.9	183.9	126.3	147.5
	8	1.25D + 0.5L - 1.4W	2,200	41.6	36.5	167.9	183.9	-126.3	-147.5
	9	0.9D + 0.5L + 1.4W	1,635	256.6	290.1	130.3	142.6	126.3	147.5
	10	0.9D + 0.5L - 1.4W	1,635	4.1	-4.8	130.3	142.6	-126.3	-147.5

2. Slenderness Effects and Sway or Nonsway Frame Designation

Columns and stories in structures are considered as nonsway frames if the stability index for the story (Q) does not exceed 0.05. ***CSA A.23.3-04 (10.14.4)***

$\sum P_f$ is the total factored vertical load in the first story corresponding to the lateral loading case for which $\sum P_f$ is greatest (without the wind loads, which would cause compression in some columns and tension in others and thus would cancel out). ***CSA A23.3-04 (10.14.4)***

V_f is the total factored shear in the first story corresponding to the wind loads, and Δ_o is the first-order relative deflection between the top and bottom of the first story due to V_f. ***CSA A.23.3-04 (10.14.4)***

From Table 1, load combination (1.25D + 1.5L) provides the greatest value of $\sum P_f$.

$$\Sigma P_f = 1.25 \times D + 1.5 \times L = 77,500 \text{ kN}$$ ***CSA A.23.3-04 (Table C.1)***

Note: Any structural analysis procedure can be performed to obtain the values of V_f and Δ_o (out of the scoop of this example).

$$V_f = 1,105 \text{ kN (given)}$$

$$\Delta_o = 7.58 \text{ mm (given)}$$

$$Q = \frac{\Sigma P_f \times \Delta_o}{V_f \times l_c} = \frac{77,500 \times 7.58}{1,105 \times 5,500} = 0.0967 > 0.05$$ ***CSA A.23.3-04 (Eq. 10-14)***

Thus, the frame at the first story level is considered sway.

3. Determine Slenderness Effects

$$I_{column} = 0.7 \times \frac{c^4}{12} = 0.7 \times \frac{500^4}{12} = 3.65 \times 10^9 \text{ mm}^4$$ **CSA A.23.3-04 (10.14.1.2)**

$$E_c = \left(3,300 \times \sqrt{f_c^{'}} + 6,900\right)\left(\frac{\gamma_c}{2,300}\right)^{1.5}$$ **CSA A.23.3-04 (Eq. 8-1)**

$$E_c = \left(3,300 \times \sqrt{25} + 6,900\right)\left(\frac{2,400}{2,300}\right)^{1.5} = 24,942.2 \text{ MPa}$$

For the column below level 2:

$$\frac{E_c \times I_{column}}{l_c} = \frac{24,942.2 \times 3.65 \times 10^9}{5,500} = 1.65 \times 10^{10} \text{ N.mm}$$

For the column above level 2:

$$\frac{E_c \times I_{column}}{l_c} = \frac{24,942.2 \times 3.65 \times 10^9}{3,500} = 2.6 \times 10^{10} \text{ N.mm}$$

For beams framing into the columns:

$$\frac{E_b \times I_{beam}}{l_b} = \frac{24,942.2 \times 5.54 \times 10^9}{9,500} = 1.45 \times 10^{10} \text{ N.mm}$$

Where:

$$E_c = \left(3,300 \times \sqrt{f_c^{'}} + 6,900\right)\left(\frac{\gamma_c}{2,300}\right)^{1.5}$$ **CSA A.23.3-04 (Eq. 8-1)**

$$E_c = \left(3,300 \times \sqrt{25} + 6,900\right)\left(\frac{2,400}{2,300}\right)^{1.5} = 24,942.2 \text{ MPa}$$

$$I_{beam} = 0.35 \times \frac{b \times h^3}{12} = 0.35 \times \frac{450 \times 750^3}{12} = 5.54 \times 10^9 \text{ mm}^4$$ **CSA A.23.3-04 (10.14.1.2)**

$$\Psi_A = \frac{\left(\sum \frac{EI}{l_c}\right)_{columns}}{\left(\sum \frac{EI}{l}\right)_{beams}} = \frac{1.65 + 2.6}{1.45} = 2.92$$ **CSA A.23.3-04 (Figure N.10.15.1)**

$\Psi_B = 1.0$ (Column considered fixed at the base) **CSA A.23.3-04 (Figure N.10.15.1)**

Using Figure N10.15.1 from CSA A23.3-04 → k = 1.51 as shown in the figure below for the exterior column.

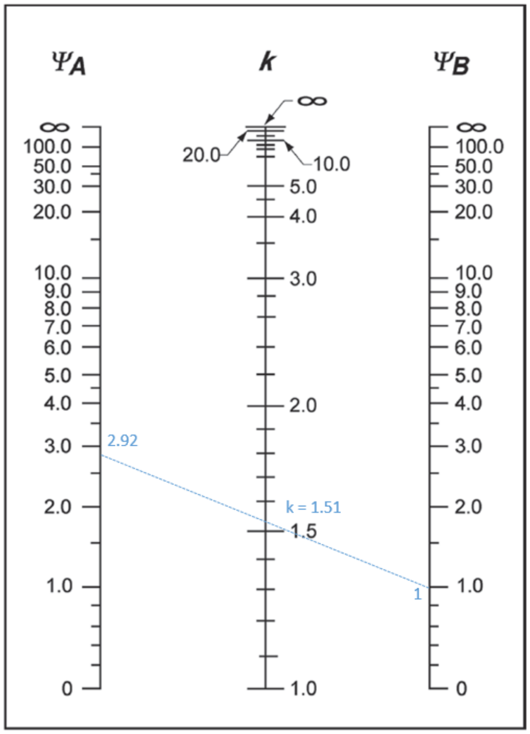


Figure 2 – Effective Length Factor (k) for Exterior Column (Sway Frame)

Note: CSA A23.3-04 (Cl. 10.15.2) allows to neglect the slenderness in a non-sway frame. However, there is no such clause in for sway frames. The CSA A23.3-04 committee intended that all columns in sway frames should be designed for slenderness.

4. Moment Magnification at Ends of Compression Member

A detailed calculation for load combinations 2 and 7 is shown below to illustrate the slender column moment magnification procedure. Table 4 summarizes the magnified moment computations for the exterior columns.

4.1. Gravity Load Combination #2 (Gravity Loads Only)

$$M_2 = M_{2ns} + \delta_s M_{2s}$$ ***CSA A23.3-04 (Eq. 10-22)***

Where:

$$M_{Top\_s} = M_{Bottom\_s} = M_{2\_s} = 0\,\text{kN.m}$$

$$\therefore M_2 = M_{2ns}$$

$$M_{Top\_2^{nd}} = M_{Top,ns} = -235.3\ \text{kN.m}$$

$$M_{Bottom\_2^{nd}} = M_{Bottom,ns} = -256.8\ \text{kN.m}$$

$$M_{2\_2^{nd}} = max\left(M_{Top\_2^{nd}}, M_{Bottom\_2^{nd}}\right) = M_{Bottom\_2^{nd}} = -256.8 \text{ kN.m} \rightarrow M_{2\_1^{st}} = M_{Bottom\_1^{st}} = -256.8 \text{ kN.m}$$

$$M_{1\_2^{nd}} = min\left(M_{Top\_2^{nd}}, M_{Bottom\_2^{nd}}\right) = M_{Top\_2^{nd}} = -235.3 \text{ kN.m} \rightarrow M_{1\_1^{st}} = M_{Top\_1^{st}} = -235.3 \text{ kN.m}$$

P_f = 2,563 kN

4.2. Lateral Load Combination #7 (Gravity Plus Wind Loads)

$$M_2 = M_{2ns} + \delta_s M_{2s}$$ ***CSA A23.3-04 (Eq. 10-22)***

Where:

$$\delta_s = \text{ moment magnifier} = \begin{Bmatrix} \text{(1) Second-order analysis} \\ \text{(2) } \dfrac{1}{1-\dfrac{\Sigma P_f}{\phi_m \Sigma P_c}} \\ \text{(3) } \dfrac{1}{1-Q}\text{, if Q < 1/3} \end{Bmatrix}$$ ***CSA A23.3-04 (10.16.3)***

There are three options for calculating δ_s. CSA A23.3-04 (10.16.3.2) will be used since it does not require a detailed structural analysis model results to proceed and is also used by the solver engine in spColumn.

$\sum P_f$ is the summation of all the factored vertical loads in the first story, and $\sum P_c$ is the summation for all sway-resisting columns in the first story.

$$P_c = \frac{\pi^2 EI}{(kl_u)^2}$$ ***CSA A23.3-04 (Eq. 10-17)***

Where:

$$EI = \begin{Bmatrix} \text{(a)} \dfrac{0.2E_c I_g + E_s I_{st}}{1+\beta_d} \\ \text{(b)} \dfrac{0.4E_c I_g}{1+\beta_d} \end{Bmatrix}$$ ***CSA A23.3-04 (10.15.3.1)***

There are two options for calculating the flexural stiffness of slender concrete columns *EI*. The first equation provides accurate representation of the reinforcement in the section and will be used in this example and is also used by the solver in spColumn. Further comparison of the available options is provided in "Effective Flexural Stiffness for Critical Buckling Load of Concrete Columns" technical note.

$$I_{column} = \frac{c^4}{12} = \frac{500^4}{12} = 5.21\times10^9 \text{ mm}^4$$ ***CSA A.23.3-04 (10.14.1.2)***

$$E_c = \left(3,300 \times \sqrt{f_c^{'}} + 6,900\right)\left(\frac{\gamma_c}{2,300}\right)^{1.5}$$ ***CSA A.23.3-04 (Eq. 8-1)***

$$E_c = \left(3,300 \times \sqrt{25} + 6,900\right)\left(\frac{2,400}{2,300}\right)^{1.5} = 24,942.2 \text{ MPa}$$

β_d in sway frames, is the ratio of maximum factored sustained shear within a story to the maximum factored shear in that story associated with the same load combination. The maximum factored sustained shear in this example is equal to zero leading to $\beta_d = 0$. ***CSA A.23.3-04 (2.3)***

For exterior columns with one beam framing into them in the direction of analysis (14 columns):

With 12 – 25M reinforcement equally distributed on all sides $I_{st} = 1.62 \times 10^8$ mm ^{4}

$$EI = \frac{0.2 E_c I_g + E_s I_{st}}{1 + \beta_d}$$ ***CSA A23.3-04 (Eq. 10-18)***

$$EI = \frac{0.2 \times 24,942.2 \times (5.21 \times 10^9) + 200,000 \times (1.62 \times 10^8)}{1+0} = 5.85 \times 10^{13} \text{ N.mm}^2$$

$k = 1.51$ (calculated previously).

$$P_{c1} = \frac{\pi^2 \times 5.85 \times 10^{13}}{(1.51 \times 4,750)^2} = 1.12 \times 10^7 \text{ N} = 11,213.9 \text{ kN}$$

For exterior columns with two beams framing into them in the direction of analysis (4 columns):

$$\Psi_A = \frac{\left(\sum \frac{EI}{l_c}\right)_{columns}}{\left(\sum \frac{EI}{l}\right)_{beams}} = \frac{1.65 + 2.6}{1.45 + 1.53} = 1.42$$ ***CSA A.23.3-04 (Figure N.10.15.1)***

$\Psi_B = 1$ (Column considered fixed at the base) ***CSA A.23.3-04 (Figure N.10.15.1)***

Using Figure N10.15.1 from CSA A23.3-04 → $k = 1.38$ as shown in the figure below for the exterior columns with two beams framing into them in the directions of analysis.

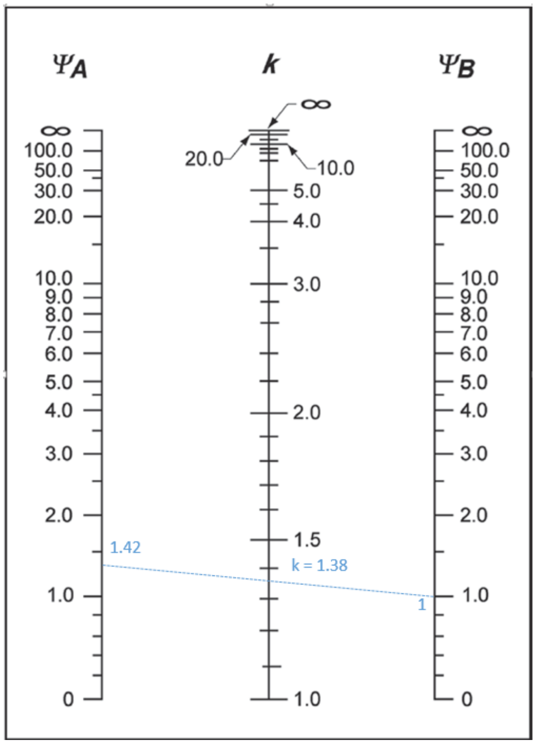


Figure 3 – Effective Length Factor (k) for Exterior Columns with Two Beams Framing into them in the Direction of Analysis

$$P_{c2} = \frac{\pi^2 \times 5.85 \times 10^{13}}{(1.38 \times 4,750)^2} = 1.34 \times 10^7 \text{ N} = 13,426.2 \text{ kN}$$

For interior columns (10 columns):

$$\Psi_A = \frac{\left(\sum \frac{EI}{l_c}\right)_{columns}}{\left(\sum \frac{EI}{l}\right)_{beams}} = \frac{1.65 + 2.6}{1.45 + 1.53} = 1.42$$ ***CSA A.23.3-04 (Figure N.10.15.1)***

$\Psi_B = 1.0$ (Column essentially fixed at base) ***CSA A.23.3-04 (Figure N.10.15.1)***

Using Figure N10.15.1 from CSA A23.3-04→ k = 1.38 as shown in the figure below for the interior columns.

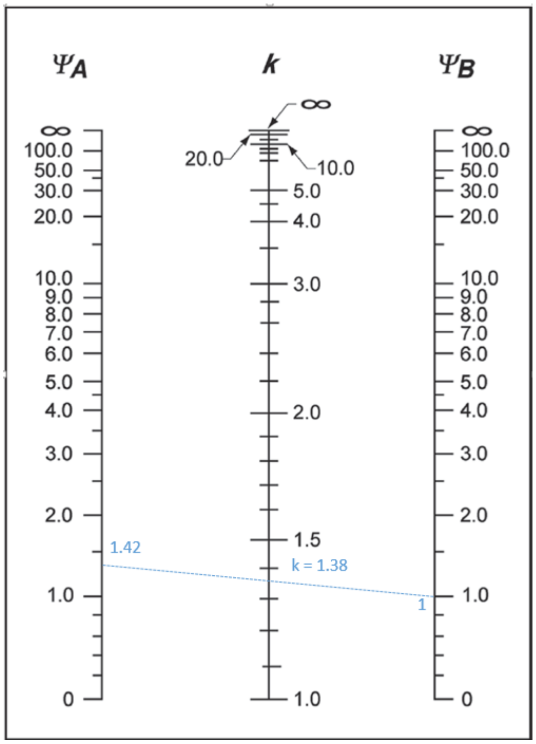


Figure 4 – Effective Length Factor (k) Calculations for Interior Columns

With 12 – 25M reinforcement equally distributed on all sides I_{st} = 1.62×10^8 mm ^{4}

$$EI=\frac{0.2E_cI_g+E_sI_{st}}{1+\beta_d}$$ ***CSA A23.3-04 (Eq. 10-18)***

$$EI=\frac{0.2\times24,942.2\times(5.21\times10^9)+200,000\times(1.62\times10^8)}{1+0}=5.85\times10^{13}\ \text{N.mm}^2$$

$$P_{c2}=\frac{\pi^2\times5.85\times10^{13}}{(1.38\times4,750)^2}=1.34\times10^7\ \text{N}=13,426.2\ \text{kN}$$

$$\Sigma P_c=n_1\times P_{c1}+n_2\times P_{c2}+n_3\times P_{c3}$$

$$\Sigma P_c=10\times13,426.2+4\times13,426.2+14\times11,213.9=344,960\ \text{kN}$$

$$\Sigma P_f=65,500\ \text{kN (Table 1)}$$

$$\delta_s=\frac{1}{1-\dfrac{\Sigma P_f}{\phi_m\Sigma P_c}}$$ ***CSA A.23.3-04 (Eq. 10-23)***

$$\delta_s=\frac{1}{1-\dfrac{65,500}{0.75\times344,960}}=1.34$$

$\delta_s M_{Top,s} = 1.34 \times 126.3 = 169.1$ kN.m

$M_{Top\_2^{nd}} = M_{Top,ns} + \delta_s M_{Top,s} = 167.9 + 169.1 = 337$ kN.m **CSA A.23.3-04 (10.16.2)**

$\delta_s M_{Bottom,s} = 1.34 \times 147.5 = 197.5$ kN.m

$M_{Bottom\_2^{nd}} = M_{Bottom,ns} + \delta_s M_{Bottom,s} = 183.9 + 197.5 = 381.4$ kN.m **CSA A.23.3-04 (10.16.2)**

$M_{2\_2^{nd}} = max\left(M_{Top\_2^{nd}}, M_{Bottom\_2^{nd}}\right) = M_{Bottom\_2^{nd}} = 381.4 \text{ kN.m} \rightarrow M_{2\_1^{st}} = M_{Bottom\_1^{st}} = 331.4$

$M_{1\_2^{nd}} = min\left(M_{Top\_2^{nd}}, M_{Bottom\_2^{nd}}\right) = M_{Top\_2^{nd}} = 337 \text{ kN.m} \rightarrow M_{1\_1^{st}} = M_{Top\_1^{st}} = 294.2 \text{ kN.m}$

P_f = 2,200 kN

A summary of the moment magnification factors and magnified moments for the exterior column for all load combinations using equation options CSA A23.3 (Eq. 10-23) to calculate δ_s is provided in the table below for illustration. Note: The designation of M_1 and M_2 is made based on the second-order (magnified) moments and not based on the first-order (unmagnified) moments.

Table 4 - Factored Axial loads and Magnified Moments at the Ends of Exterior Column

No.	Load Combination	Axial Load	Using CSA Eq. 10-23		
		kN	δ_s	M_1, kN.m	M_2, kN.m
1	1.4D	2,261	---	150.3	165.2
2	1.25D + 1.5L	2,563	---	235.3	256.8
3	1.25D + 1.5L + 0.4W	2,563	1.43	286.8	316.9
4	1.25D + 1.5L - 0.4W	2,563	1.43	183.8	196.6
5	0.9D + 1.5L + 0.4W	1,998	1.31	244.9	270.6
6	0.9D + 1.5L - 0.4W	1,998	1.31	150.6	160.4
7	1.25D + 0.5L + 1.4W	2,200	1.34	337.0	381.4
8	1.25D + 0.5L - 1.4W	2,200	1.34	-1.2	-13.5
9	0.9D + 0.5L + 1.4W	1,635	1.23	286.0	324.4
10	0.9D + 0.5L - 1.4W	1,635	1.23	-25.3	-39.1

5. Moment Magnification along Length of Compression Member

In sway frames, if an individual compression member has:

$$\frac{l_u}{r} > \frac{35}{\sqrt{P_f / (f'_c A_g)}}$$ **CSA A23.3-04 (Eq. 10-25)**

It shall be designed for the factored axial load, P_f and moment, M_c, computed using Clause 10.15.3 (Nonsway frame procedure), in which M_1 and M_2 are computed in accordance with Clause 10.16.2. **CSA A23.3-04 (10.16.4)**

$$M_c = \frac{C_m M_2}{1 - \dfrac{P_f}{\phi_m P_c}} \geq M_2$$ **CSA A23.3-04 (10.15.3.1)**

Where:

$$C_m = 0.6 + 0.4\frac{M_1}{M_2} \geq 0.4$$ **CSA A23.3-04 (10.15.3.2)**

M_2 = the second-order factored moment (magnified sway moment)

And, the member resistance factor would be $\phi_m = 0.75$ **CSA A23.3-04 (10.15.3.1)**

$$P_c = \frac{\pi^2 EI}{(kl_u)^2}$$ **CSA A23.3-04 (Eq. 10-17)**

Where:

$$EI = \begin{cases} \text{(a)} \ \dfrac{0.2E_c I_g + E_s I_{st}}{1+\beta_d} \\ \text{(b)} \ \dfrac{0.4E_c I_g}{1+\beta_d} \end{cases}$$ **CSA A23.3-04 (10.15.3.1)**

There are two options for calculating the effective flexural stiffness of slender concrete columns *EI*. The first equation provides accurate representation of the reinforcement in the section and will be used in this example and is also used by the solver in spColumn. Further comparison of the available options is provided in "Effective Flexural Stiffness for Critical Buckling Load of Concrete Columns" technical note.

5.1. Gravity Load Combination #2 (Gravity Loads Only)

$$r = \sqrt{\frac{I_g}{A_g}} = \sqrt{\frac{500^4/12}{500^2}} = 144.34 \text{ mm}$$ **CSA A23.3-04 (10.14.2)**

$$\frac{l_u}{r} = \frac{4,750}{144.34} = 32.91$$

$$\frac{35}{\sqrt{P_f/(f_c^{'}A_g)}} = \frac{35}{\sqrt{\dfrac{2,564\times1,000}{25\times2.5\times10^5}}} = 54.64$$ ***CSA A23.3-04 (Eq. 10-25)***

Since 32.94 < 54.64, calculating the moments along the column length is not required.

Check minimum moment: ***CSA A23.3-04 (10.15.3)***

CSA A23.3-04 does not require to design columns in sway frames for a minimum moment. However, the reference decided conservatively to design the column for the larger of computed moments and the minimum value of M_2.

$$(M_2)_{min} = P_f(15+0.03h)$$

$$(M_2)_{min} = 2,563\times(15+0.03\times500)/1,000 = 76.9 \text{ kN.m}$$

5.2. Load Combination #7 (Gravity Plus Wind Loads)

$$\frac{35}{\sqrt{P_f/(f_c^{'}A_g)}} = \frac{35}{\sqrt{\dfrac{2,200\times1,000}{25\times2.5\times10^5}}} = 58.99$$ ***CSA A23.3-04 (Eq. 10-25)***

Since 32.94 < 56.48, calculating the moments along the column length is not required.

Check minimum moment: ***CSA A23.3-04 (10.15.3.1)***

$$(M_2)_{min} = P_f(15+0.03h)$$

$$(M_2)_{min} = 2,200\times(15+0.03\times500)/1,000 = 66 \text{ kN.m}$$

M_{c1} and M_{c2} will be considered separately to ensure proper comparison of resulting magnified moments against negative and positive moment capacities of unsymmetrical sections as can be seen in the following figure.

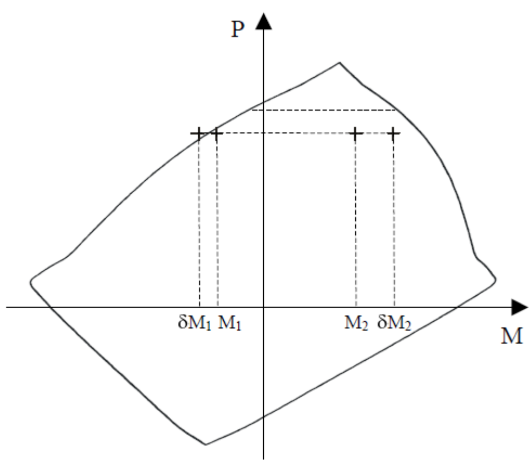


Figure 5 – Column Interaction Diagram for Unsymmetrical Section

A summary of the moment magnification factors and magnified moments for the exterior column for all load combinations using equation CSA A23.3 (Eq. 10-23) to calculate δ_s is provided in the table below for illustration.

Table 5 - Factored Axial loads and Magnified Moments along Exterior Column Length					
No.	Load Combination	Axial Load, kN	Using CSA Eq. 10-23		
			δ	M_{c1}, kN.m	M_{c2}, kN.m
1	1.4D	2,261	1	150.3	165
2	1.25D + 1.5L	2,563	1	235.3	256.8
3	1.25D + 1.5L + 0.4W	2,563	1	286.8	316.9
4	1.25D + 1.5L - 0.4W	2,563	1	183.8	196.7
5	0.9D + 1.5L + 0.4W	1,998	1	245	270.5
6	0.9D + 1.5L - 0.4W	1,998	1	150.6	160.5
7	1.25D + 0.5L + 1.4W	2,200	1	337	381.4
8	1.25D + 0.5L - 1.4W	2,200	1	-1.2	-13.6
9	0.9D + 0.5L + 1.4W	1,635	1	421.3	471.9
10	0.9D + 0.5L - 1.4W	1,635	1	100.9	108.3

For column design, CSA A23.3 requires that δ_s to be computed from Clause 10.16.3.2 using $\sum P_f$ and $\sum P_c$ under gravity load shall be positive and shall not exceed 2.5. β_d shall be taken as the ratio of the maximum factored sustained axial load to the maximum factored axial load associated with the same load combination. For values of δ_s above the limit, the frame would be very susceptible to variations in EI, foundation rotations and the like. If this value exceeds 2.5, the frame must be stiffened to reduce δ_s. ***CSA A23.3-04 (10.16.5 & N10.16.5)***

$$\beta_d = \frac{\text{Maximum factored sustained axial load}}{\text{Maximum factored axial load (same load combination)}}$$ ***CSA A23.3-04 (10.16.5)***

$$\beta_d = \frac{66{,}640}{66{,}640} = 1$$

$$P_c = \frac{\pi^2 \mathrm{EI}}{(\mathrm{kl_u})^2}$$ ***CSA A23.3-04 (Eq. 10-17)***

Where:

$$EI = \frac{0.2E_c I_g + E_s I_{st}}{1+\beta_d}$$ ***CSA A23.3-04 (Eq. 10-18)***

$$EI = \frac{0.2\times 24{,}942.2\times(5.21\times10^9)+200{,}000\times(1.62\times10^8)}{1+1} = 2.92\times10^{13}\ \text{N.mm}^2$$

For exterior columns with two beams framing into them in the direction of analysis:

$$P_c = \frac{\pi^2\times 2.92\times10^{13}}{(1.51\times 4{,}750)^2} = 5{,}606.9\ \text{kN}$$

For interior columns and exterior columns with two beams framing into them in the direction of analysis:

$$P_c = \frac{\pi^2 \times 2.92 \times 10^{13}}{(1.38 \times 4{,}750)^2} = 6{,}713.1 \text{ kN}$$

$$\Sigma P_c = (10+4) \times 6{,}713.1 + 14 \times 5{,}606.9 = 172{,}480.3 \text{ kN}$$

Where the member resistance factor is $\phi_m = 0.75$ **_CSA A23.3-04 (10.15.3.1)_**

$$\delta_s = \frac{1}{1 - \frac{\Sigma P_f}{\phi_m \times \Sigma P_c}}$$ **_CSA A23.3-04 (Eq. 10-23)_**

$$\delta_s = \frac{1}{1 - \frac{66{,}640}{0.75 \times 172{,}480.3}} = 2.06 < 2.5$$

Thus, the frame is stable.

6. Column Design

Based on the factored axial loads and magnified moments considering slenderness effects, the capacity of the assumed column section (500 mm × 500 mm with 12 – 25M bars distributed all sides equal) will be checked and confirmed to finalize the design. A column interaction diagram will be generated using strain compatibility analysis, the detailed procedure to develop column interaction diagram can be found in "Interaction Diagram - Tied Reinforced Concrete Column" example.

The factored axial load resistance P_r for all load combinations will be set equals to P_f, then the factored moment resistance M_r associated to P_r will be compared with the magnified applied moment M_f. The design check for load combination #7 is shown below for illustration. The rest of the checks for the other load combinations are shown in the following Table.

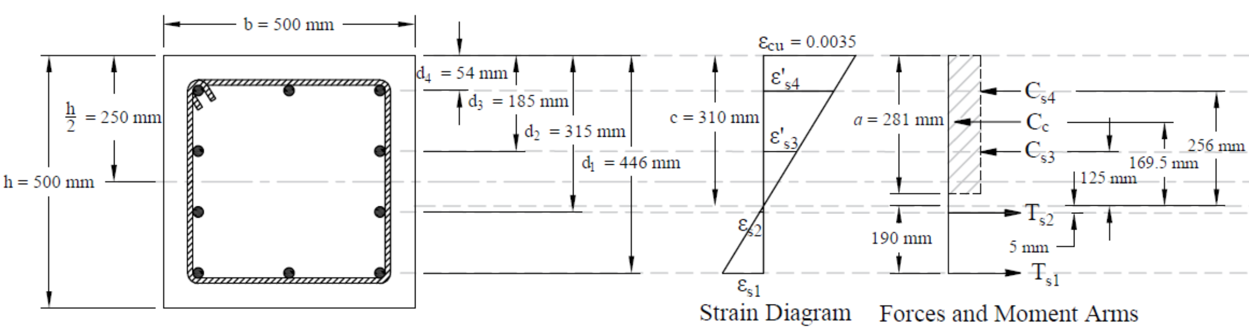


Figure 6 – Strains, Forces, and Moment Arms (Load Combination 7)

The following procedure is used to determine the nominal moment capacity by setting the factored axial load resistance, P_r, equal to the factored axial load, P_f and iterating on the location of the neutral axis.

6.1. c, *a*, and strains in the reinforcement

Try $c = 310$ mm

Where c is the distance from extreme compression fiber to the neutral axis. ***CSA A.23.3-04 (2.3)***

$a = \beta_1 \times c = 0.908 \times 310 = 281 \text{ mm}$ ***CSA A.23.3-04 (10.1.7a)***

Where:

$\beta_1 = 0.97 - 0.0025 f_c^{'} = 0.908 \geq 0.67$ ***CSA A.23.3-04 (Eq. 10-2)***

$\varepsilon_{cu} = 0.0035$ ***CSA A.23.3-04 (10.1.3)***

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{400}{200,000} = 0.002$$

$$\varepsilon_s = (d_1 - c) \times \frac{0.0035}{c} = (446 - 310) \times \frac{0.0035}{310} = 0.00154 \text{ (Tension)} < \varepsilon_y$$

$\therefore$ tension reinforcement has not yielded

$\phi_c = 0.65$ ***CSA A.23.3-04 (8.4.2)***

$\phi_s = 0.85$ ***CSA A.23.3-04 (8.4.3)***

$$\varepsilon_{s4}^{'} = (c - d_4) \times \frac{0.0035}{c} = (310 - 54) \times \frac{0.0035}{310} = 0.00289 \text{ (Compression)} > \varepsilon_y$$

$$\varepsilon_{s3}^{'} = \left(c - d_3\right) \times \frac{0.0035}{c} = (310 - 185) \times \frac{0.0035}{310} = 0.00141 \text{ (Compression)} < \varepsilon_y$$

$$\varepsilon_{s2} = \left(d_2 - c\right) \times \frac{0.0035}{c} = (315 - 310) \times \frac{0.0035}{310} = 5.65 \times 10^{-5} \text{ (Tension)} < \varepsilon_y$$

6.2. Forces in the concrete and steel

$C_{rc} = \alpha_1 \times \phi_c \times f_c^{'} \times a \times b = 0.812 \times 0.65 \times 25 \times 281 \times 500 = 1,857.2 \text{ kN}$ ***CSA A.23.3-04 (10.1.7a)***

Where:

$\alpha_1 = 0.85 - 0.0015 f_c^{'} = 0.812 \geq 0.67$ ***CSA A.23.3-04 (Eq. 10-1)***

$f_s = \varepsilon_s \times E_s = 0.00154 \times 200,000 = 307.1 \text{ MPa}$

$\text{T}_{rs1} = \phi_s \times f_s \times A_{s1} = 0.85 \times 307.1 \times \left(4 \times 500\right) = 552.1 \text{ kN}$

Since $\varepsilon_{s4}^{'} > \varepsilon_y \rightarrow$ compression reinforcement has yielded

$\therefore f_{s4}^{'} = f_y = 400 \text{ MPa}$

Since $\varepsilon_{s3}^{'} < \varepsilon_y \rightarrow$ compression reinforcement has not yielded

$$\therefore\ f_{s3}^{'} = \varepsilon_{s3}^{'} \times E_s = 0.00141 \times 200{,}000 = 282.3\ \text{MPa}$$

Since $\varepsilon_{s2} < \varepsilon_y \rightarrow$ tension reinforcement has not yielded

$$\therefore\ f_{s2} = \varepsilon_{s2} \times E_s = 5.65 \times 10^{-5} \times 200{,}000 = 11.3\ \text{MPa}$$

The area of the reinforcement in third and fourth layers has been included in the area (*ab*) used to compute C_{rc}. As a result, it is necessary to subtract $\alpha_1\ f_c'$ from f_s' before computing C_{rs}:

$$\text{C}_{rs4} = \left(\phi_s\, f_{s4}^{'} - \alpha_1\, \phi_c\, f_c^{'}\right) \times A_{s4}^{'} = \left(0.85 \times 400 - 0.812 \times 0.65 \times 25\right) \times \left(4 \times 500\right) / 1{,}000 = 653.6\ \text{kN}$$

$$\text{C}_{rs3} = \left(\phi_s\, f_{s3}^{'} - \alpha_1\, \phi_c\, f_c^{'}\right) \times A_{s3}^{'} = \left(0.85 \times 282.3 - 0.812 \times 0.65 \times 25\right) \times \left(2 \times 500\right) / 1{,}000 = 226.7\ \text{kN}$$

$$\text{T}_{rs2} = \left(\phi_s\, f_{s2}^{'}\right) \times A_{s2}^{'} = \left(0.85 \times 11.3\right) \times \left(2 \times 500\right) / 1{,}000 = 9.6\ \text{kN}$$

6.3. P_r and M_r

$$P_r = C_{rc} + C_{rs3} + C_{rs4} - T_{rs1} - T_{rs2} = 1{,}857.2 + 226.7 + 653.6 - 552.1 - 9.6 = 2{,}205.8\ \text{kN}$$

$$P_r = 2{,}205.8\ \text{kN} \simeq 2{,}200\ \text{kN} = P_f$$

The assumed value of c = 310 mm is correct.

$$M_r = C_{rc} \times \left(\frac{h}{2} - \frac{a}{2}\right) + C_{rs4} \times \left(\frac{h}{2} - d_4\right) + C_{rs3} \times \left(\frac{h}{2} - d_3\right) + T_{rs2} \times \left(d_2 - \frac{h}{2}\right) + T_{rs1} \times \left(d_1 - \frac{h}{2}\right)$$

$$M_r = 1{,}857.2 \times \left(\frac{500}{2} - \frac{281}{2}\right) + 653.6 \times \left(\frac{500}{2} - 54\right) + 226.7 \times \left(\frac{500}{2} - 185\right) + 9.6 \times \left(315 - \frac{500}{2}\right) + 522.1 \times \left(446 - \frac{500}{2}\right)$$

$$M_r = 448{,}849\ \text{N.m} = 448.8\ \text{kN.m} > M_f = 381.4\ \text{kN.m}$$

Table 6 – Exterior Column Axial and Moment Capacities						
No.	P_f, kN	$M_u = M_{2(2nd)}$, kN.m	c, mm	$\varepsilon_t = \varepsilon_s$	P_r, kN	M_r, kN.m
1	2,261	165	314	0.00147	2,263.8	443.6
2	2,563	256.8	336	0.00115	2,568.3	414.9
3	2,563	316.9	336	0.00115	2,568.3	414.9
4	2,563	196.7	336	0.00115	2,568.3	414.9
5	1,998	270.5	296	0.00177	1,998	467.6
6	1,998	160.5	296	0.00177	1,998	467.6
7	2,200	381.4	310	0.00154	2,205.8	448.8
8	2,200	-13.6	310	0.00154	2,205.8	448.8
9	1,635	471.9	267	0.00235	1,635.7	485.5
10	1,635	108.3	267	0.00235	1,635.7	485.5

Since $M_r > M_f$ for all $P_r = P_f$, use 500 × 500 mm column with 12 – 25M bars.

7. Column Interaction Diagram - spColumn Software

spColumn program performs the analysis of the reinforced concrete section conforming to the provisions of the Strength Design Method and Unified Design Provisions with all conditions of strength satisfying the applicable conditions of equilibrium and strain compatibility and includes slenderness effects using moment magnification method for sway and nonsway frames. For this column section, we ran in investigation mode with control points using the CSA A23.3-04. In lieu of using program shortcuts, spSection (Figure 7) was used to place the reinforcement and define the cover to illustrate handling of irregular shapes and unusual bar arrangement.

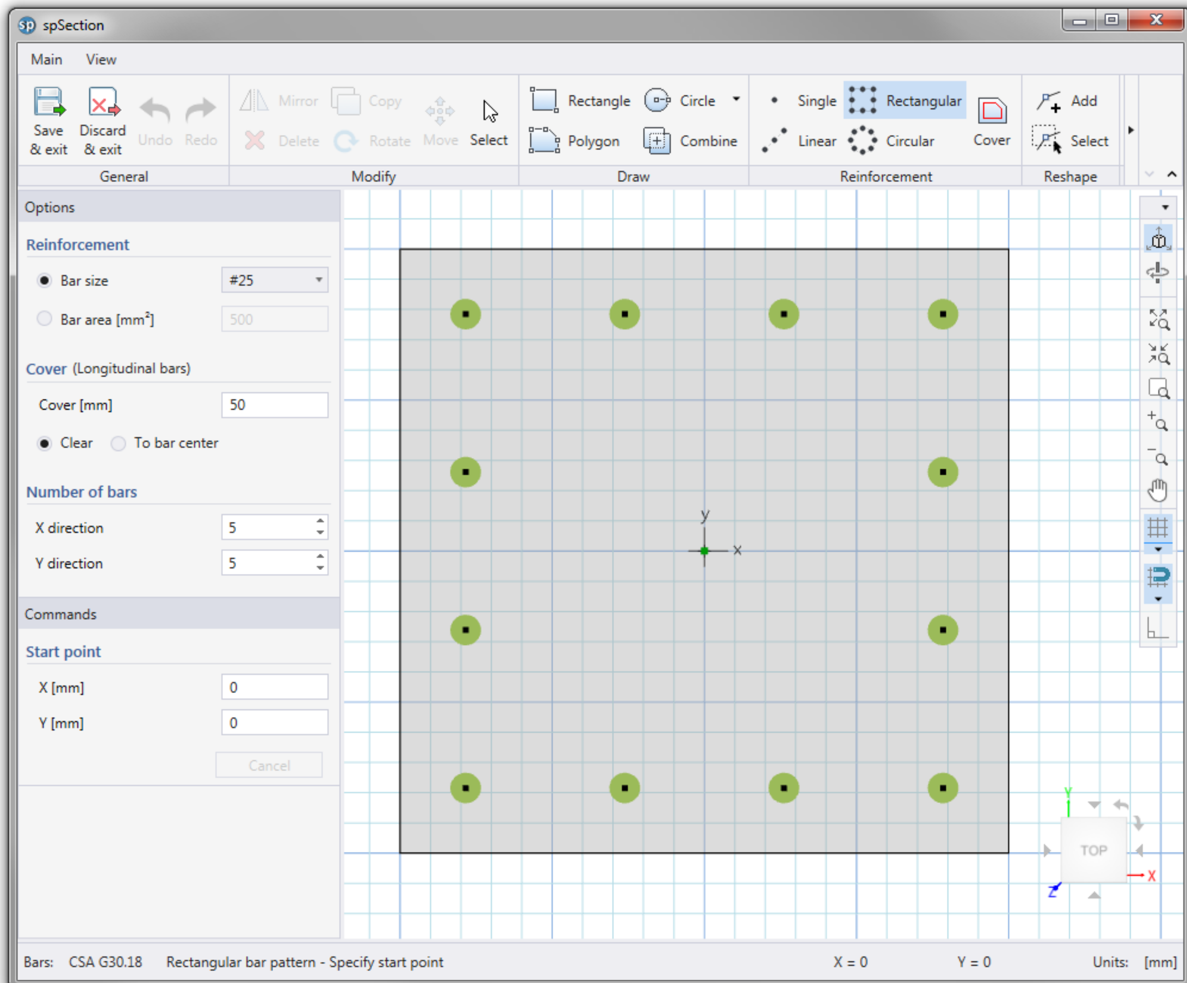


Figure 7 – spColumn Model Editor (spSection)

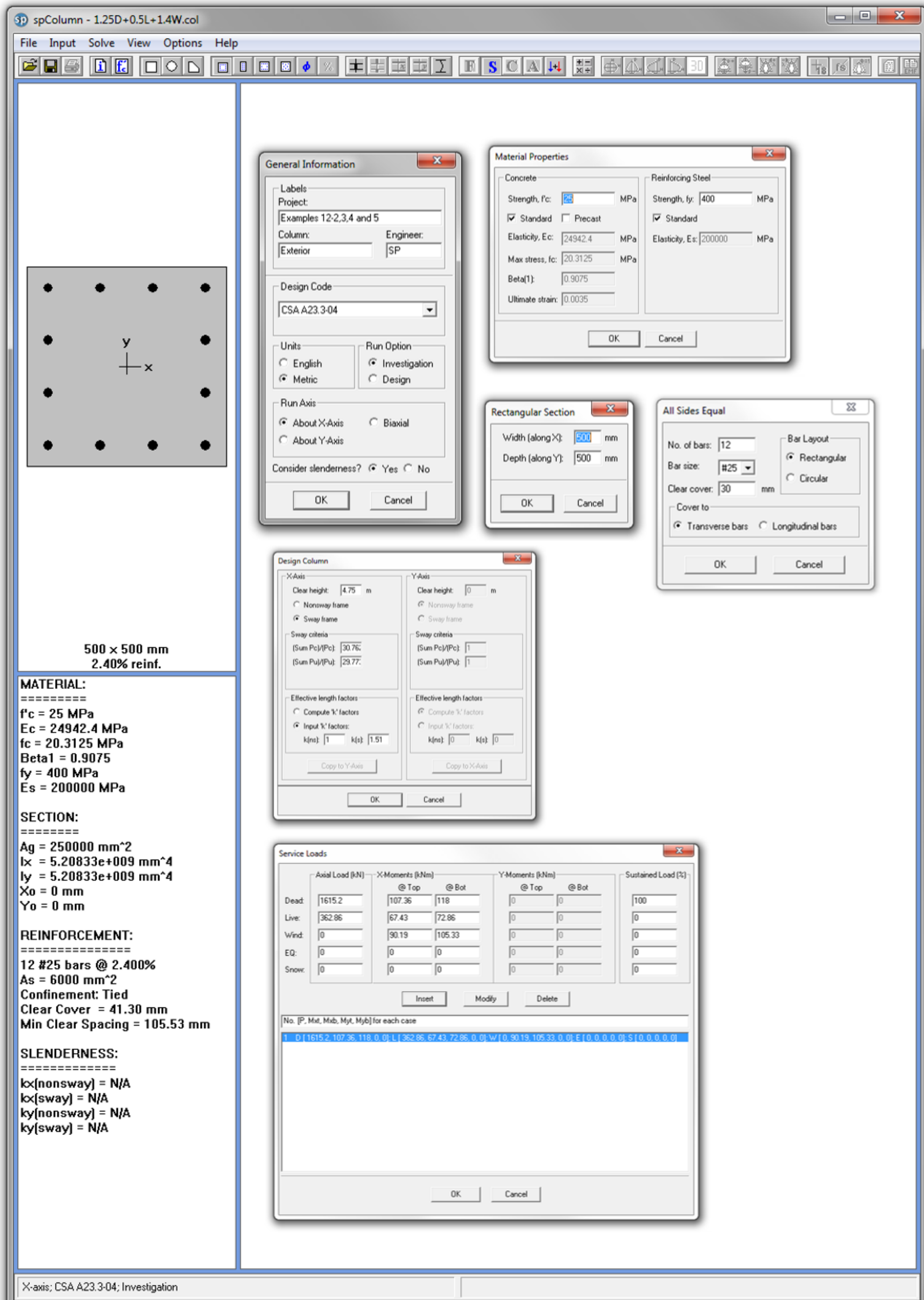
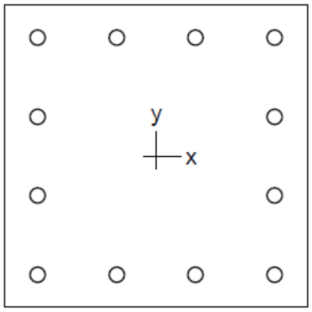


Figure 8 –spColumn Model Input Wizard Windows



500 x 500 mm

Code: CSA A23.3-04

Units: Metric

Run axis: About X-axis

Run option: Investigation

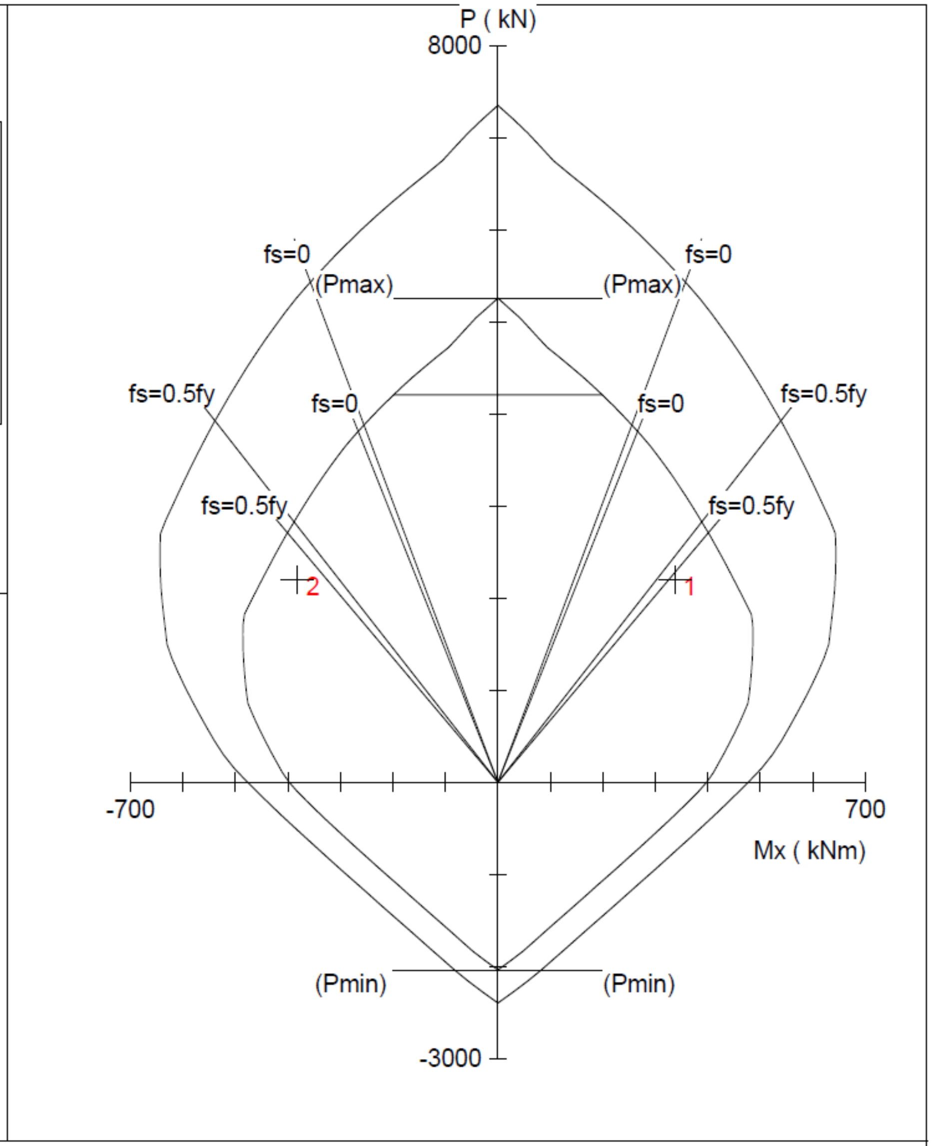
Slenderness: Considered

Column type: Structural

Bars: CSA G30.18

Date: 06/19/18

Time: 14:41:17



STRUCTUREPOINT - spColumn v6.00 (TM). Licensed to: StructurePoint. License ID: 00000-0000000-4-25EF2-2C6B6

File: C:\TSDA\Slenderness\CSA A23.3-04\1.25D+0.5L+1.4W.col

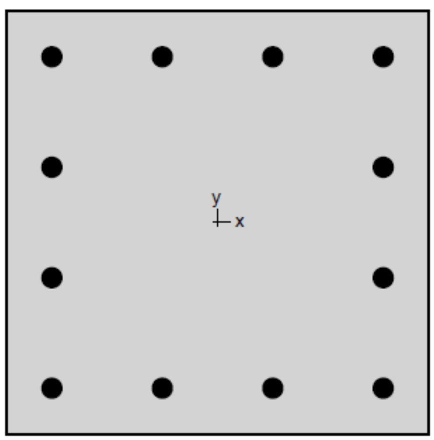
Project: Examples 12-2,3,4 and 5

Column: Exterior		Engineer: SP	
f'c = 25 MPa	fy = 400 MPa	Ag = 250000 mm^2	12 #25 bars
Ec = 24942 MPa	Es = 200000 MPa	As = 6000 mm^2	rho = 2.40%
fc = 20.3125 MPa	e_yt = 0.002 mm/mm	Xo = 0 mm	Ix = 5.21e+009 mm^4
e_u = 0.0035 mm/mm		Yo = 0 mm	Iy = 5.21e+009 mm^4
Beta1 = 0.9075		Min clear spacing = 106 mm	Clear cover = 41 mm
Confinement: Tied		kx(nonsway) = 1	kx(sway) = 1.51

phi(a) = 0.8, phi(s) = 0.85, phi(c) = 0.65, phi(rho) = 1.00

Figure 5 – Column Section Interaction Diagram about X-Axis – Design Check for Load Combination 7 (spColumn)

spColumn v6.00
Computer program for the Strength Design of Reinforced Concrete Sections



Contents

List of Figures

1. General Information

File Name	C:\TSDA\Slenderness\CSA A2...\1.25D+0.5L+1.4W.col
Project	Examples 12-2,3,4 and 5
Column	Exterior
Engineer	SP
Code	CSA A23.3-04
Bar Set	CSA G30.18
Units	Metric
Run Option	Investigation
Run Axis	X - axis
Slenderness	Considered
Column Type	Structural

2. Material Properties

2.1. Concrete

Type	Standard
f'_c	25 MPa
E_c	24942.4 MPa
f_c	20.3125 MPa
ε_u	0.0035 mm/mm
β_1	0.9075

2.2. Steel

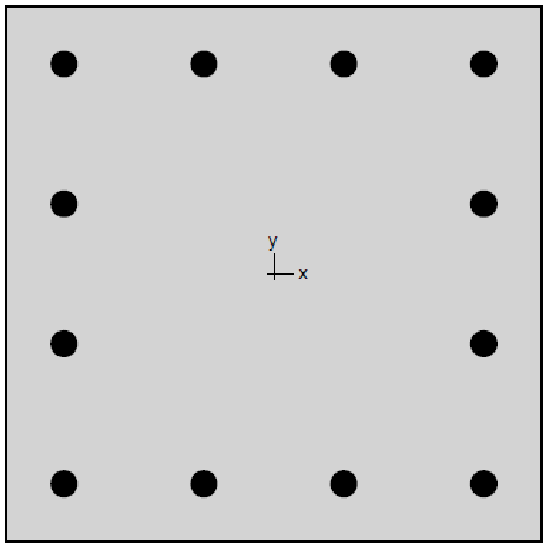
Type	Standard
f_y	400 MPa
E_s	200000 MPa
ε_{yt}	0.002 mm/mm

3. Section

3.1. Shape and Properties

Type	Rectangular
Width	500 mm
Depth	500 mm
A_g	250000 mm²
I_x	5.20833e+009 mm⁴
I_y	5.20833e+009 mm⁴
r_x	144.338 mm
r_y	144.338 mm
X_o	0 mm
Y_o	0 mm

3.2. Section Figure



Rectangular 500 x 500 mm 2.40% reinf.

Figure 1: Column section

4. Reinforcement

4.1. Bar Set: CSA G30.18

Bar	Diameter mm	Area mm^2	Bar	Diameter mm	Area mm^2	Bar	Diameter mm	Area mm^2
#10	11.30	100.00	#15	16.00	200.00	#20	19.50	300.00
#25	25.20	500.00	#30	29.90	700.00	#35	35.70	1000.00
#45	43.70	1500.00	#55	56.40	2500.00			

4.2. Confinement and Factors

Confinement type	Tied
For #55 bars or less	#10 ties
For larger bars	#15 ties
Material Resistance Factors	
Axial compression, (a)	0.8
Steel (ϕ_s)	0.85
Concrete (ϕ_c)	0.65

4.3. Arrangement

Pattern	All sides equal
Bar layout	Rectangular
Cover to	Transverse bars
Clear cover	30 mm
Bars	12 #25

Total steel area, A_s	6000 mm²
Rho	2.40 %
Minimum clear spacing	106 mm

5. Loading

5.1. Load Combinations

Combination	Dead	Live	Wind	EQ	Snow
U1	1.250	0.500	1.400	0.000	0.000

5.2. Service Loads

No.	Load Case	Axial Load	Mx @ Top	Mx @ Bottom	My @ Top	My @ Bottom
		kN	kNm	kNm	kNm	kNm
1	Dead	1615.20	107.36	118.00	0.00	0.00
1	Live	362.86	67.43	72.86	0.00	0.00
1	Wind	0.00	90.19	105.33	0.00	0.00
1	EQ	0.00	0.00	0.00	0.00	0.00
1	Snow	0.00	0.00	0.00	0.00	0.00

5.3. Sustained Load Factors

Load Case	Factor
	%
Dead	100
Live	0
Wind	0
EQ	0
Snow	0

6. Slenderness

6.1. Sway Criteria

X-Axis	Sway column
$\sum P_c$	30.76 x P_c
$\sum P_u$	29.77 x P_u

6.2. Columns

Column	Axis	Height	Width	Depth	I	f'c	Ec
		m	mm	mm	mm⁴	MPa	MPa
Design	X	4.75	500	500	5.20833e+009	25	24942.4
Above	X	(no column specified...)					
Below	X	(no column specified...)					

6.3. X - Beams

Beam	Length	Width	Depth	I	f'c	Ec
	m	mm	mm	mm⁴	MPa	MPa
Above Left	(no beam specified...)					
Above Right	(no beam specified...)					
Below Left	(no beam specified...)					
Below Right	(no beam specified...)					

7. Moment Magnification

7.1. General Parameters

Factors	Code defaults

Stiffness reduction factor, ϕ_K	0.75
Cracked section coefficients, cI(beams)	0.35
Cracked section coefficients, cI(columns)	0.7
$0.2\ E_c\ I_g + E_s\ I_{se}$ (X-axis)	5.85e+010 kNmm²
Minimum eccentricity, $e_{x\,min}$	30.00 mm
k'	$(P_f / (f'_c{*}A_g))^{0.5}$

7.2. Effective Length Factors

Axis	Ψ_{top}	Ψ_{bottom}	k (Nonsway)	k (Sway)	kl_u/r
X	0.000	0.000	1.000	1.510	49.69

7.3. Magnification Factors: X - axis

Load Combo		At Ends					Along Length					
		$\sum P_f$ kN	P_c kN	$\sum P_c$ kN	β_{ds}	δ_s	P_f kN	$k'l_u/r$	P_c kN	β_{dns}	C_m	δ
1	U1	65513.40	11214.50	344980.56	0.000	1.339	2200.43	19.53	(N/A)	(N/A)	(N/A)	(N/A)

8. Factored Moments

NOTE: Each loading combination includes the following cases:
Top - At column top
Bot - At column bottom

8.1. X - axis

Load Combo			1st Order				2nd Order			Ratio
			M_{ns} kNm	M_s kNm	M_f kNm	M_{min} kNm		M_i kNm	M_c kNm	$2^{nd}/1^{st}$
1	U1	Top	167.91	126.27	294.18	(N/A)	M_1=	336.99	(N/A)	(N/A)
1	U1	Bot	-183.93	-147.46	-331.39	(N/A)	M_2=	-381.39	(N/A)	(N/A)

9. Factored Loads and Moments with Corresponding Capacities

NOTE: Each loading combination includes the following cases:
Top - At column top
Bot - At column bottom

No.	Load Combo			P_f kN	M_{fx} kNm	M_{rx} kNm	M_r/M_f	NA Depth mm	d_t Depth mm	ε_t
1	1	U1	Top	2200.43	336.99	449.68	1.334	310	446	0.00154
2	1	U1	Bot	2200.43	-381.39	-449.68	1.179	310	446	0.00154

8. Summary and Comparison of Design Results

Analysis and design results from the hand calculations above are compared for the one load combination used in the reference (Example 12-3,4 and 5) and exact values obtained from spColumn model.

Table 7 – Parameters for Moment Magnification at Column Ends					
	k	EI, N.mm^2	P_c, kN	$M_{1(2nd)}$, kN.m	$M_{2(2nd)}$, kN.m
Hand	1.51	5.85×10^{13}	11,214	337	381.4
spColumn	1.51	5.85×10^{13}	11,214	337	381.4

In this table, a detailed comparison for all considered load combinations are presented for comparison.

Table 8 - Factored Axial loads and Magnified Moments at Column Ends								
No.	P_f, kN		δ_s		$M_{1(2nd)}$, kN.m		$M_{2(2nd)}$, kN.m	
	Hand	spColumn	Hand	spColumn	Hand	spColumn	Hand	spColumn
1	2,261.3	2,261.3	N/A	N/A	150.3	150.3	165.2	165.2
2	2,563.3	2,563.3	N/A	N/A	235.3	235.3	256.8	256.8
3	2,563.3	2,563.3	1.43	1.43	286.8	286.8	316.9	316.9
4	2,563.3	2,563.3	1.43	1.43	183.8	183.8	196.6	196.6
5	1,998.0	1,998	1.31	1.31	244.9	244.9	270.6	270.6
6	1,998.0	1,998	1.31	1.31	150.6	150.6	160.4	160.4
7	2,200.4	2,200.4	1.34	1.34	337.0	337	381.4	381.4
8	2,200.4	2,200.4	1.34	1.34	-1.2	-1.2	-13.5	-13.5
9	1,635.1	1,635	1.23	1.23	286.0	286	324.4	324.4
10	1,635.1	1,635	1.23	1.23	-25.3	-25.3	-39.1	-39.1

Table 9 - Design Parameters Comparison								
No.	c, mm		$\varepsilon_t = \varepsilon_s$		P_f, kN		M_r, kN.m	
	Hand	spColumn	Hand	spColumn	Hand	spColumn	Hand	spColumn
1	314	314	0.00147	0.00147	2,263.8	2,261.3	443.6	444
2	336	336	0.00147	0.00147	2,568.3	2,563.3	414.9	415.7
3	336	336	0.00115	0.00115	2,568.3	2,563.3	414.9	415.7
4	336	336	0.00115	0.00115	2,568.3	2,563.3	414.9	415.7
5	296	296	0.00177	0.00177	1,995.6	1,998	467.6	467.7
6	296	296	0.00177	0.00177	1,995.6	1,998	467.6	467.7
7	310	310	0.00154	0.00154	2,200.4	2,200.4	448.8	449.7
8	310	310	0.00154	0.00154	2,200.4	2,200.4	448.8	449.7
9	267	267	0.00235	0.00235	1,635.1	1,635	485.5	485.8
10	267	267	0.00235	0.00235	1,635.1	1,635	485.5	485.8

All the results of the hand calculations illustrated above are in precise agreement with the automated exact results obtained from the spColumn program.

9. Conclusions & Observations

The analysis of the reinforced concrete section performed by spColumn conforms to the provisions of the Strength Design Method and Unified Design Provisions with all conditions of strength satisfying the applicable conditions of equilibrium and strain compatibility and includes slenderness effects using moment magnification method for sway and nonsway frames.

CSA A23.3 provides multiple options for calculating values of *EI* and δ_s leading to variability in the determination of the adequacy of a column section. Engineers must exercise judgment in selecting suitable options to match their design condition. The spColumn program utilizes the exact methods whenever possible and allows user to override the calculated values with direct input based on their engineering judgment wherever it is permissible.

It was concluded in the CSA A23.3-04 that the probability of stability failure increases rapidly when the stability index Q exceeds 0.2 and a more rigid structure may be required to provide stability. ***CSA A23.3-04 (10.14.6)***

If a frame undergoes appreciable lateral deflections under gravity loads, serious consideration should be given to rearranging the frame to make it more symmetrical because with time, creep will amplify these deflections leading to both serviceability and strength problems. One of these limitations is to limit the second-order lateral deflections to first-order lateral deflections to 2.5 (the ratio should not exceed 2.5) under factored gravity load plus a lateral load applied to each story equal to 0.0005 multiplied by factored gravity load in that story.

CSA A23.3-04 (10.16.5 & N10.16.5)

The limitation on δ_s is intended to prevent instability under gravity loads alone. For values of δ_s above the limit, the frame would be very susceptible to variations in EI, foundation rotations and the like. If δ_s exceeds 2.5 the frame must be stiffened to reduce δ_s. ***CSA A23.3-04 (N10.16.5)***

Exploring the impact of other code permissible equation options provides the engineer added flexibility in decision making regarding design. In some cases resolving the stability concern may be viable through a frame analysis providing values for V_f and Δ_o to calculate magnification factor δ_s. Creating a complete model with detailed lateral loads and load combinations to account for second order effects may not be warranted for all cases of slender column design nor is it disadvantageous to have a higher margin of safety when it comes to column slenderness and frame stability considerations.